Sonic Crystals: Fundamentals, characterization and experimental techniques

A. Cebrecos

1Laum, Le Mans Université, CNRS, Av. O. Messiaen, 72085, Le Mans

Collaborators in this work:

LAUM: J.P. Groby, V. Romero-García
UPV: N. Jiménez, V. Sánchez-Morcillo, L.M. García-Raffi
UCB: M. Hussein, D. Krattiger
Outline

• Part I
  ◦ Introduction to Sonic Crystals
    ◦ Origins and fundamentals
    ◦ Bandgaps. Dispersion relation.
      ◦ Group and phase velocities.
      ◦ Dispersion.
    ◦ EFC
    ◦ Methods for Dispersion Relation calculation
      ◦ Plane Wave expansion
      ◦ Finite-elements for band structure calculation in time-domain

• Part II
  ◦ Experimental techniques for the characterization of Sonic Crystals
    ◦ Dispersion relation using Space-time to frequency-wavevector transformation
      ◦ Methodology
    ◦ 2D Experimental relation using SLatCow
    ◦ Deconvolution method for analysis of reflected fields by SCs
      ◦ Methodology
      ◦ Practical application: Sonic Crystals for noise reduction at the launch pad
A crystal is a solid material whose constituents, such as atoms, molecules or ions, are arranged in a highly ordered microscopic structure, forming a lattice that extends in all directions.

**Artificial (or even natural) materials whose physical properties are periodic functions of the space (1D, 2D, 3D)**

**Optical properties (electromagnetic waves)**
- Dielectric constant (refractive index)

**Photonic crystal**
- Engineer photonic density of states to control the spontaneous emission of materials embedded in the photonic crystal.
- Using photonic crystals to affect localization and control of light.

**Elastic properties (elastic and acoustic waves)**
- Density and elastic constants (speed of sound)

**Phononic crystal**
- Periodic distribution of solid scatterers in a solid host medium

**Sonic crystal**
- Particular case in which the host medium is a fluid.
- M. S. Kushwaha. APL, 70, 3218. (1997)
- J. V. Sánchez-Pérez, PRL, 80, 5325. (1998)

The study of photonic, phononic and sonic crystals makes use of the same concepts and theories developed in quantum mechanics for electron motion:
- Direct and reciprocal lattices, Brillouin zone, Bragg interferences, Bloch periodicity, band structures...
• Considering that $\vec{a}_i$ are the vectors defining the lattice $\vec{R}$ in $\mathbb{R}^n$ with $i=1, \ldots, n$, thus $\vec{R}$ could be defined as

$$\vec{R} = \sum v_i \vec{a}_i$$

Where $v_i \in \mathbb{Z}$. The parallelepiped defined by the vectors forms the well known **primitive cell**, which is a particular kind of unit cell.

• There is a unique one-dimensional (1D) periodic system.
• Five two-dimensional (2D).
• Fourteen three-dimensional (3D) different lattices

**Geometrical parameters**

• Unit cell and lattice constant $a$
• Filling fraction
• Scatterer shape

$$f_f = \frac{A_{sc}}{A_{uc}}$$
Reciprocal Space

- The set of wave vectors $\vec{k}$ that yield plane waves with the periodicity of a given direct lattice is known as its reciprocal lattice.

- For a set of primitive vectors in the direct lattice, the reciprocal lattice is generated:

\[
\vec{b}_i = \frac{\epsilon_{ijk} \vec{a}_j \times \vec{a}_k}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}
\]

- where $\epsilon_{ijk}$ is the completely anti-symmetric Levi-Civita symbol.

Properties:

- Orthogonality: $\vec{a}_i \cdot \vec{b}_j = 2\pi \delta_{ij}$

- Any linear combination $\vec{k} = \sum_{i=1}^{n} \mu_i \vec{b}_i$ with $\mu_i \in \mathbb{Z}$ reaches a point of the reciprocal lattice.

The reciprocal space (k-space) is used to study the wave propagation characteristics in periodic structures (Dispersion relation, bandgaps, propagation direction):
Reciprocal Space

- The set of wave vectors \( \vec{k} \) that yield plane waves with the periodicity of a given direct lattice is known as its reciprocal lattice.

- For a set of primitive vectors in the direct lattice, the reciprocal lattice is generated:

  \[
  \vec{b}_i = \frac{\varepsilon_{ijk} \vec{a}_j \times \vec{a}_k}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}
  \]

- where \( \varepsilon_{ijk} \) is the completely anti-symmetric Levi-Civita symbol.

Properties:

- Orthogonality: \( \vec{a}_i \cdot \vec{b}_j = 2\pi \delta_{ij} \)

- Any linear combination of reciprocal lattice vectors

  \[
  \vec{k} = \sum_{i=1}^{n} \mu_i \vec{b}_i \quad \text{with} \quad \mu_i \in \mathbb{Z}, \text{ reaches a point of the reciprocal lattice.}
  \]

The study of the band structure can be limited to the Irreducible Brillouin zone. For the case of a square lattice the IBZ is reduced to a triangle. The scatterer should accomplish the same symmetries in the direct space.
Direct and Reciprocal Space

- **Direct space**
  - Propagation pressure fields, Vibrational modes

- **Reciprocal space**
  - Band structures, Equifrequency contours, surfaces

**ORIGINS AND FUNDAMENTALS**
Origins and interpretation of the band gaps: Bragg interferences

- The path difference between the two incident rays is $2d \sin \theta$, where $\theta$ is the angle of incidence.
- **Bragg condition:** $2d \sin \theta = n\lambda$ $\Rightarrow$ $\lambda = 2d$
- **N** is the order of the corresponding reflection.

J. D. Joannopoulos (Photonic crystals: Molding the flow of light)

$\kappa + i \kappa; \ e^{-\kappa x}$

Illustration of Evanescent behaviour, 1D

Evidence of evanescent modes inside the BG

- The path difference between the two incident rays is \(2d \sin \theta\), where \(\theta\) is the angle of incidence
- Bragg condition: \(2d \sin \theta = n\lambda\) \(\Rightarrow\) \(\lambda = 2d\)
- \(N\) is the order of the corresponding reflection

Analytical, numerical and experimental results

- Analytical value: \(Im(k) = -5.6 \text{ m}^{-1}; \nu = 920 \text{ Hz}\)
- Experimental value: \(Im(k) = -5.60 \pm 1.45 \text{ m}^{-1}; \nu = 920 \text{ Hz}\)

Origins of the band gaps

- $w(k)$ in homogeneous medium

Plane wave propagating in a 1D homogeneous medium
Origins of the band gaps

- Periodicity in real space
- Periodicity in reciprocal space

Plane wave propagating in a 1D homogeneous medium

\[ p(x + a) = p(x), \]

\[ k = k_0 + \frac{2\pi n}{a}. \]

\[ \omega = ck \]
Origins of the band gaps

- Periodic $w(k)$ repetitions of period $2\pi/a$ (replicated bands due to periodicity)

Plane wave propagating in a 1D homogeneous medium

\[ p(x + a) = p(x), \]
\[ k = k_0 + \frac{2\pi n}{a}. \]

\[ \omega = ck \]
\[ \omega = ck \]
Origins of the band gaps

- Periodic variation of the physical properties of the medium

**Plane wave propagating in a 1D periodic medium**

\[
p(x + a) = p(x),
\]

\[
k = k_0 + \frac{2\pi n}{a}.
\]

\[
\omega = ck,
\]

\[
-\frac{\pi}{a} \quad +\frac{\pi}{a}
\]
Origins of the band gaps

- Creation of bandgaps and dispersion

Plane wave propagating in a 1D periodic medium

\[ p(x + a) = p(x), \]

\[ k = k_0 + \frac{2\pi n}{a}. \]

\[ \omega = c(\omega)k \]

\[ \omega = ck \]
Origins of the band gaps

- Band gap frequency and dependence of its width

**Plane wave propagating in a 1D periodic medium**

**Band gap depends on:**

- Periodicity
  \[ k_{BG} = \frac{2\pi\nu_{BG}}{c} = \frac{\pi}{a} \]
  \[ \nu_{BG} = \frac{c}{2a} \]

- Filling fraction:
  \[ \Delta\omega = f(\text{ff}) \]

- Contrast
2D Dispersion relation

- $w(k)$ in 2D is now a surface

Plane wave propagating in a 2D homogeneous medium

\[
\omega = c \sqrt{k_x^2 + k_y^2}
\]
Periodicity in x and y directions. Main directions of symmetry in reciprocal space.

Plane wave propagating in a 2D homogeneous medium.

\[
p(x + R) = p(x) \\
R = na\hat{i} + ma\hat{j}
\]

\[
k_x = \left[ -\frac{\pi}{a}, \frac{\pi}{a} \right], \quad k_y = \left[ -\frac{\pi}{a}, \frac{\pi}{a} \right]
\]

\[
\left( \frac{\omega}{c} \right)^2 = k_x^2 + k_y^2
\]

\[
R^2 = x^2 + y^2
\]
2D Dispersion relation

- Dispersion relation along the main symmetry points of the reciprocal space

Plane wave propagating in a 2D homogeneous medium

\[ \omega = c \sqrt{k_x^2 + k_y^2} \]
2D dispersion relation

- Periodic variation of the physical properties. Creation of pseudo gaps for low ff

Plane wave propagating in a 2D periodic medium
2D dispersión relation

- Increasing ff and/or impedance contrast will eventually create a full band gap

Plane wave propagating in a 2D periodic medium
Phase and group velocities

- **Phase velocity**: The velocity at which the phase of a wave of frequency $w$ and wavevector $k$, propagates:
  \[ v_p = \frac{w}{k} \]

- **Group velocity**: The velocity at which a wave packet containing several wave vectors and frequencies travels:
  \[ v_g = \frac{\partial w}{\partial k} \]

- In lossless dispersive media $v_g$ represents the velocity of the wave packet and the energy direction.

- In a lossless non dispersive media, $v_g = v_p$
Group velocity and phase velocity

- Negative group velocity: Certain frequencies in the second band

\[ v_g = \nabla_k (\omega(k)) \]
\[ v_p = \frac{\omega}{k} \]

Source: Institute of Sound and Vibration Research. University of Southampton
Group velocity and phase velocity

- Higher phase velocity: Dispersive part of first band

\[ v_g = \nabla_k (\omega(k)) \]
\[ v_p = \frac{\omega}{k} \]

Source: Institute of Sound and Vibration Research. University of Southampton
Group velocity and phase velocity

- Zero group velocity: Localized wave

\[ v_g = \nabla_k (\omega(k)) \]
\[ v_p = \frac{w}{k} \]

Source: Institute of Sound and Vibration Research. University of Southampton.
Understanding dispersion

- Group velocity: \( v_g = \nabla_k (\omega(k)) \)
- Phase velocity: \( v_p = \frac{w}{k} \)

Plane wave propagating in a 1D periodic medium

N = 200 layers
Rigid boundary conditions
Gaussian pulse at \( w_n = 0.4 \)
Understanding dispersion

- **Group velocity**  
  \[ v_g = \nabla_k (\omega(k)) \]

- **Phase velocity**  
  \[ v_p = \frac{w}{k} \]

Plane wave propagating in a 1D periodic medium

N = 200 layers  
Rigid boundary conditions  
Gaussian pulse at \( w_n = 0.7 \)
Understanding dispersion

- Group velocity $v_g = \nabla_k (\omega(k))$
- Phase velocity $v_p = \frac{w}{k}$

N = 200 layers
Rigid boundary conditions
Gaussian pulse at $w_n = 1$

Plane wave propagating in a 1D periodic medium
• Equi-frequency contours (EFC) offer rich information about wave propagation in periodic media. They represent the intersection of a constant frequency $w$-plane to a dispersion surface.
  - Not existent in 1D. In 2D they are lines. In 3D they are surfaces.
  - Group velocity vector is perpendicular to the EFC, $\nabla_k (w(k))$, represents the energy direction.
Generally considering infinite media by applying periodic boundary conditions (Bloch theory)

**PERIODIC : INFINITE MEDIUM**

- Transfer matrix method (TMM)
- Plane wave expansion (PWE)
- Multiple scattering method (MST)
- Finite element method (FEM)
- Finite difference in time domain (FDTD)
- Finite element in time domain (FETD)

**CHARACTERISTICS**

- Dimensionality: 1D, 2D, 3D (TMM 1D, PWE, FEM, FDTD all, etc.)
- Handling different media (limitations in PWE solid-fluid except rigid inclusions in air or holes in solid matrix)
- Handling geometry (structure factor in PWE, meshing in FEM or FDTD)
- Steady-state or time dependent
PWE. Eigenvalue problem

• Plane wave expansion (PWE)

Wave equation (inhomogeneous medium)

\[ \frac{1}{B(\vec{r})} \frac{\partial^2 p}{\partial t^2} = \nabla \left( \frac{1}{\rho(\vec{r})} \nabla p \right) \]

Considering a periodic medium

**Fourier series expansion**

\[
\sigma = \frac{1}{\rho(\vec{r})} = \sum_{\vec{G}} \sigma^k_{\vec{G}}(\vec{G}) e^{i\vec{G}\cdot\vec{r}}
\]

\[
\eta = \frac{1}{B(\vec{r})} = \sum_{\vec{G}} \eta^k_{\vec{G}}(\vec{G}) e^{i\vec{G}\cdot\vec{r}}
\]

**Bloch’s theorem**

The solution of a wave equation with periodic potential can be written in the form:

\[ p(\vec{r}, t) = e^{i(\vec{k}\cdot\vec{r} - \omega t)} p(\vec{r}) \]

With \( p(\vec{r}) \) a function that possesses the periodicity of the direct lattice, therefore

\[ p(\vec{r}, t) = e^{i(\vec{k}\cdot\vec{r} - \omega t)} \sum_{\vec{G}} p^k_{\vec{G}}(\vec{G}) e^{i\vec{G}\cdot\vec{r}} \]
Model the shape of the scatterer:

\[
\sigma_k(\vec{G}) = \frac{1}{A_{uc}} \int_{A_{uc}} \sigma(\vec{r}) e^{-i \vec{G} \cdot \vec{r}} dS = \frac{1}{A_{uc}} \left[ \int_{A_s} \sigma_A e^{-i \vec{G} \cdot \vec{r}} dS + \int_{A_{uc}-A_s} \sigma_B e^{-i \vec{G} \cdot \vec{r}} dS \right]
\]

\[
\sigma_k(\vec{G}) = \begin{cases} 
\sigma_A f f + \sigma_B (1 - f f) & \text{if } \vec{G} = \vec{0} \\
(\sigma_A - \sigma_B) F(\vec{G}) & \text{if } \vec{G} \neq \vec{0}
\end{cases}
\]

\[
F(\vec{G}) = \frac{1}{A_{uc}} \int_{A_s} e^{-i \vec{G} \cdot \vec{r}} d\vec{r}
\]

Where \( F(\vec{G}) \) is the structure factor, \( A_s \) and \( A_{uc} \) are the surfaces of the scatterer and of the unit cell, respectively, so the filling fraction is \( f f = A_s / A_{uc} \)

Kushwaha et al., PRB, 49. (1994)

Cylinder

\[
F(\vec{G}) = \frac{2 ff}{Gr_0} J_1(Gr_0)
\]

\( J_1 \) is the Bessel function of the first kind and \( r_0 \) the radius of the cylinder

Square

\[
\text{Square-rod scatterers of side } l \text{ rotated } \theta
\]

\[
F(\vec{G}, \theta) = f \text{sinc} \left( \frac{G_x l}{2} \right) \text{sinc} \left( \frac{G_y l}{2} \right)
\]

V. Romero-Garcia. JPD, 40. 305108. (2013)

Other shapes

Hexagonal, rectangular, elliptic

R. Wang et al, JAP, 90, 2001
PWE. Eigenvalue problem

\[ \sum_{\vec{G}'} \left( (\vec{k} + \vec{G}) \sigma_{\vec{k}} (\vec{G} - \vec{G}') (\vec{k} + \vec{G}') - \omega^2 \eta_{\vec{k}} (\vec{G} - \vec{G}') \right) p_{\vec{k}}(\vec{G}') = 0 \]

**w(\vec{k}) method**

\[ \sum_{i=1}^{3} \Gamma_i \Sigma \Gamma_i P = \omega^2 \Omega P \]

Where \( i = 1, 2, 3 \). The matrices \( \Gamma_i \), Reciprocal vectors \( \Sigma \), Density \( \Omega \), Bulk modulus

Solving this system of equations for each Bloch vectors in the irreducible Brillouin zone, we obtain \( N \times N \) eigenvalues, \( w^2 \), which can be used to represent the band structures, \( w(\vec{k}) \)

- Dispersion relation of a SC made of rigid cylinders embedded in air.
- Lattice constant \( a = 4.6 \) cm
- radius \( r = 1 \) cm.
- filling fraction is \( ff = \pi r^2/a^2 = 0.068 \)
Finite element method in time-domain

- Finite element in time-domain for elastic band structure calculation (FETD)
  - Time-dependent Bloch periodicity
  - Excitation required

- Continuum Equation of motion:

\[
\nabla \cdot \sigma = \rho \ddot{u} \quad \nabla \cdot C : \nabla^S u = \rho \ddot{u}
\]


**Strong form general elastodynamic problem**

**Space Discretization**
Unit-cell finite-element model

- Weak form:

\[- \int_{\Omega} \left( \nabla^S w : C : \nabla^S u \right) \, d\Omega = \int_{\Omega} (\rho \dot{w} \cdot \ddot{u}) \, d\Omega \]

\[\Omega = \bigcup_{e=1}^{n_l} \Omega^e\]

Weighting and shape functions

\[w_1 = N_A w_{1A}, \quad A = 1, n_{en}\]

\[u = N_B d_{1B}, \quad B = 1, n_{en}\]

\[- \int_{\Omega} \left( \nabla^S N_A w_{1A} : C : \nabla^S N_B d_{1B} \right) \, d\Omega = \int_{\Omega} \left( \rho N_A w_{1A} \cdot N_B \ddot{d}_{1B} \right) \, d\Omega.\]

\[-M \ddot{U} + KU = 0\]

Direct stiffness method

Element to global

\[M = \sum_{e=1}^{n_{el}} M^e \quad K = \sum_{e=1}^{n_{el}} K^e\]

Option: Eigenvalue problem

or time-integration (forced term)

Bloch theory (BC’s)

\[u(x, k; t) = \tilde{u}(x, k) e^{i(k^T x - \omega t)}\]

FINITE-ELEMENTS FOR BAND STRUCTURE CALCULATION IN TIME-DOMAIN
Unit-cell time-domain simulation

- Time integration method:

\[ \mathbf{D}_{n+1} = \mathbf{D}_i + \Delta t \mathbf{V}_i + (\Delta t)^2 \left[ \left( \frac{1}{2} - \beta \right) \mathbf{A}_i + \beta \mathbf{A}_{i+1} \right], \]

\[ \mathbf{V}_{i+1} = \mathbf{V}_i + \Delta t \left[ (1 - \gamma) \mathbf{A}_i + \gamma \mathbf{A}_{i+1} \right], \]

\[ \mathbf{M} \mathbf{A}_{i+1} + \mathbf{K} \mathbf{D}_{i+1} = \mathbf{F}_{i+1} \]

\[ \beta = 0 \quad \gamma = 1/2 \]

- Center difference Newmark scheme (Explicit)
  - Computationally efficient
  - Less storage compared to implicit methods
  - Conditionally stable

CFL

\[ \Delta t = \frac{\Delta h^e}{c_{\text{max}}} \]
Unit-cell time-domain simulation

- Transient excitation
  - Ricker wavelet
    \[ u(t) = a^2 \left[ a^2 t^2 - 1 \right] e^{-\frac{a^2 t^2}{2}} \]

- Calculation of frequency band structure
Unit-cell time-domain simulation

- Transient excitation
  - Ricker wavelet
    \[ u(t) = a^2 \left( a^2 t^2 - 1 \right) e^{-\frac{a^2 t^2}{2}} \]

- Calculation of frequency band structure
Numerical examples. Bloch Modeshapes

FINITE-ELEMENTS FOR BAND STRUCTURE CALCULATION IN TIME-DOMAIN
Dispersion relation using Space-time to frequency-wavevector transformation
1D Dispersion relation recovery: Methodology

- Space-Time Fourier Transformation

\[ p(x, t) \xrightarrow{\text{Space-Time Fourier Transformation}} p(k, w) \]

1D periodic medium

\[ h(x_1, t) \]

\[ h(x_2, t) \]
1D Dispersion relation recovery

- Spatial sampling

\[ p(x, t) \rightarrow p(k, w) \]

1D periodic medium

\[ p(x, f_1) \]

\[ p(x, f_2) \]
What is the right spatial sampling measuring periodic structures?

Using analogy from temporal signals:

\[ p(x) \]

\[ k_s = \frac{1}{dx} \]

\[ dk = \frac{1}{L_s} = \frac{1}{dxN} \]

From Nyquist sampling theorem, considering periodicity in reciprocal space:

\[ k_{max} \leq 2\pi \frac{k_s}{2} \]

\[ k_{max} = \pi / a \]

Finally, consider a SC of a given length:

\[ L_{sc} = a \cdot N_{uc} \]

For smaller \( dx \), greater \( N \)

\[ dk = \frac{1}{dxN} = \frac{1}{aN_{uc}} \]

\( N^\circ \) UNIT CELLS DEFINES RESOLUTION in \( k \)-space
1D Dispersion relation recovery

- Influence of SC size, undersampling and oversampling space

\[ N_{uc} = 10 \quad d x = a \]

\[ N_{uc} = 50 \quad d x = a \]

Number of Unit cells

\[ N_{uc} = 50 \quad d x = 2a \]

Undersampling

\[ N_{uc} = 50 \quad d x = a/3 \]

Oversampling

DISPERSION RELATION USING SPACE-TIME TO FREQUENCY-WAVEVECTOR TRANSFORMATION
2D Dispersion relation recovery: Methodology

- 2D Acoustic metamaterial
  - Quarter-wave resonators as scatterers (Lossless)
  - 50 x 50 unit cells
  - 50 measurement points in center line

\[ dx = a \]

- Point source at the center

\[ \vec{k}_x \neq 0 \quad \vec{k}_y \neq 0 \]

Angular components in all directions

Bands overlapping

Dispersion relation incomplete!
2D Dispersion relation recovery: Methodology

- 2D Acoustic metamaterial
  - Quarter-wave resonators as scatterers (Lossless)
  - 50 x 50 unit cells
  - 50 measurement points in center line
  
  \[ dx = a \]

- Plane wave excitation
  
  \[ \vec{k}_x \neq 0 \quad \vec{k}_y = 0 \]

New angular components created due to scattering

\[ \vec{k}_y \neq 0 \]

Dispersion relation incomplete!
2D Dispersion relation recovery: Methodology

- Spatial “filtering” in the reciprocal space (\(k\)-space)
  
  1. 1 measurement point per scatterer
      
      \[ dx = dy = a \]

  2. Transformation to \(k\)-space (2D)
      
      \[ p(x, y, t) \rightarrow p(k_x, k_y, \omega) \rightarrow \text{Isofrequency contours} \]

  3. Selection of main symmetry directions for frequencies of interest
      
      \[ X' \Gamma \quad \Gamma X \quad XM \quad \Gamma M \]
2D Dispersion relation recovery: Methodology

- Spatial “filtering” in the reciprocal space (\(k\)-space)
- Results using a point source
2D Dispersion relation recovery: Methodology

- Spatial “filtering” in the reciprocal space ($k$-space)
- Results using a plane wave
Experimental dispersion relation measurement

- SLaTCow method: Complex dispersion relation

\[ \mathcal{L}[p(x, w)] \rightarrow p(k_r, k_i, w) \]

\[ \text{Time domain FT} \]

\[ p(x, t) \rightarrow p(x, w) \]

\[ \text{Spatial domain Laplace Transform} \]

Analysis for every frequency

- Set of parameters defining theoretically the propagation: amplitude, phase, real and imaginary part of the wave vector

\[ \zeta_{th}(\{\zeta\}) \]

- Optimization minimizing the difference between \( \mathcal{L}[\zeta_{th}] \) and \( \mathcal{L}[\zeta_{exp}] \)

\[ \{\zeta\} = \text{argmin} \Delta(\{\zeta'\}) \]

\[ \Delta(\{\zeta'\}) = \sqrt{\sum_{s} |\mathcal{L}[\zeta_{exp}](s) - \mathcal{L}[\zeta_{th}(\{\zeta'\})](s)|^2} \]

A. Geslain et al., J. Appl. Phys. 120, 135107, (2016) Wednesday morning at SAPEM
Experimental dispersion relation measurement

- SLaTCow method: Complex dispersion relation

\[ p(x, t) \overset{\text{Time domain FT}}{\longrightarrow} p(x, w) \]
\[ \mathcal{L}[p(x, w)] \overset{\text{Spatial domain Laplace Transform}}{\longrightarrow} p(k_r, k_i, w) \]

Analysis for every frequency

- Set of parameters defining theoretically the propagation: amplitude, phase, real and imaginary part of the wave vector

\[ \zeta_{th}(\{\zeta\}) \]

- Optimization minimizing the difference between \( \mathcal{L}[\zeta_{th}] \) and \( \mathcal{L}[\zeta_{exp}] \)

\[ \{\zeta\} = \text{argmin} \Delta(\{\zeta'\}) \]
\[ \Delta(\{\zeta'\}) = \sqrt{\sum_s |\mathcal{L}[\zeta_{exp}](s) - \mathcal{L}[\zeta_{th}(\{\zeta'\})](s)|^2} \]
Experimental dispersion relation

- 24 x 4 unit cells
- 1 measurement point per scatterer

Numerical results

Experiments
Experimental dispersion relation

- 24 x 4 unit cells
- 1 measurement point per scatterer
Experimental dispersion relation

- 24 x 4 unit cells
- 1 measurement point per scatterer

$c_i$

$f = 711 \text{ Hz}$
$f = 1777 \text{ Hz}$

$f = 2975 \text{ Hz}$
$f = 2056 \text{ Hz}$
Deconvolution method for analysis of reflected fields by SCs

PART II
Experimental evaluation of a SC Impulse Response

• Context of the problem:
  ◦ Study feasibility of Sonic Crystals in reducing impact of the backward reflected field emitted by a sound source in a simplified scaled model of a launch pad (Proof of concept)

• Requisites (Greatly simplified problem):
  ◦ Linear regime
  ◦ Static source
  ◦ Broadband frequency study
  ◦ Scaled model in water (ultrasonic regime)

• Challenge:
  ◦ Analysis of reflected field by a SC
    ◦ Insertion loss in reflection
    ◦ Diffusion coefficient

ESA – ITI Type A project: Sonic Crystals for Noise Reduction at the Launch Pad
Experimental evaluation of a SC Impulse Response

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ESA – ITI Type A project: Sonic Crystals for Noise Reduction at the Launch Pad

DECONVOLUTION METHOD FOR ANALYSIS OF REFLECTED FIELDS BY SCS
Linear and time invariant system: Theory

• A linear and time-invariant system is fully characterized by its impulse response (IR), $h(t)$.

\[ y(t) = x(t) \ast h(t) \quad Y(f) = X(f) \ast H(f) \]

• In acoustics experiments (and many others types) it is usual to work in the frequency domain

Transfer function $H(f)$:
• Widely used in experiments using SC’s
• Calculation of reflection, transmission and absorption coefficients
• Insertion Loss in attenuation devices
Linear and time invariant system: Theory

- A linear and time-invariant system is fully characterized by its impulse response (IR), $h(t)$.

$$y(t) = x(t) * h(t) \quad \quad Y(f) = X(f) * H(f)$$

- Impulse response by deconvolution (input and output known):

$$x^{-1}(t) * y(t) = x^{-1}(t) * [x(t) * h(t)] = \delta(t) * h(t)$$

$$h(t) = x^{-1}(t) * y(t)$$

Impulse response $h(t)$:

- Room acoustics: Acoustic quality parameters
- Weakly nonlinear system identification
- Incident and reflected pressure field


Impulse response deconvolution: Practical example

- Input signal: Logarithmic sine sweep:
  \[ x(t) = \sin \left( \frac{2\pi f_1 T}{\ln \frac{f_2}{f_1}} (e^{-t/L} - 1) \right) \]

- Quasi-ideal case 1: \( y(t) = x(t) \)

\[
\begin{align*}
X^{-1}(f) & \quad Y(f) & \quad H(f) \\
\text{Frequency (Hz)} & \times 10^6 & \text{Frequency (Hz)} & \times 10^6 & \text{Time (s)} & \times 10^{-5}
\end{align*}
\]

\[ f_1 = 500 \text{ Hz} \]
\[ f_2 = 10 \text{ MHz} \]
\[ T = 50 \text{ ms} \]
\[ f_s = 20 \text{ MHz} \]
- Input signal: Logarithmic Sine sweep:

\[ x(t) = \sin \left( \frac{2\pi f_1 T}{\ln \frac{f_2}{f_1}} (e^{-t/L} - 1) \right) \]

- Quasi-ideal case 2: \( y(t) = x(t) \)

\[ f_1 = 200 \text{ kHz} \]
\[ f_2 = 800 \text{ kHz} \]
\[ T = 1 \text{ ms} \]
\[ f_s = 20 \text{ MHz} \]
Experimental setup

- Signal parameters
  \[ f_1 = 200 \text{ kHz} \quad T = 1 \text{ ms} \]
  \[ f_2 = 800 \text{ kHz} \quad f_s = 20 \text{ MHz} \]

Computer.
Labview for measurement control

DAQ-PXI
Digital Signal Generator
Digital oscilloscope

3D Motorized axis

Hydrophone
Piezoelectric transducer
RF Amplifier
Input signal: Logarithmic Sine sweep:

\[ x(t) = \sin \left( \frac{2\pi f_1 T}{\ln \frac{f_2}{f_1}} (e^{-t/L} - 1) \right) \]

Real case:
- System response + reflections \( y(t) \neq x(t) \)
Impulse response deconvolution: Example

- **Input signal**: Logarithmic Sine sweep:
  \[
  x(t) = \sin \left( \frac{2\pi f_1 T}{\ln \frac{f_2}{f_1}} (e^{-t/L} - 1) \right)
  \]

- **Real case**:
  - System response + reflections 
    \[ y(t) \neq x(t) \]

\[ x^{-1}(t) \]

\[ y(z_1, t) \]

\[ h(z_1, t) \]

\[ y(z_2, t) \]

\[ h(z_2, t) \]
Samples used in the experiments

- **Characteristics of the SCs**
  - Four different samples (2D SCs)
    - Square shape scatterers
    - Square and triangular lattice
    - Low and filling fraction
  - 3D Printed
    - Selective laser melting
Incident and reflected field separation

- Experimental results: Incident field

\[ f = 350 \text{ kHz} \]
First Band

\[ f = 500 \text{ kHz} \]
Band Gap

\[ f = 700 \text{ kHz} \]
Second Band

DECONVOLUTION METHOD FOR ANALYSIS OF REFLECTED FIELDS BY SCS
Incident and reflected field separation

- Reflected field. Square lattice High filling fraction

\[ f = 700 \text{ kHz} \]
High Band

\[ f = 350 \text{ kHz} \]
Low Band

\[ f = 500 \text{ kHz} \]
Band Gap
Experimental wave propagation. Convolution signal on demand

- **Input:** Sinusoidal pulse (Narrow bandwidth, band gap) \( f_c = 500 \text{ kHz} \)

Supplementary Video 1

**Reflector** \( t = 200 \mu \text{s} \)

**SC-SH** \( t = 200 \mu \text{s} \)

**Reflector,** \( h(x=0, z=200) \)

**SC-SH,** \( h(x=0, z=200) \)
Experimental wave propagation

- Input: Sinusoidal pulse (Narrow bandwidth, 2° Band) $f_c = 660$ kHz

**Supplementary Video 3**

**Reflector, $t = 200 \mu s$**

**SC-TL, $t = 200 \mu s$**
Insertion loss results

- Modified Insertion loss to study reflection

\[
IL \,[\text{dB}] = 10 \log_{10} \left( \frac{|P_{\text{ref}}|^2}{|P_{\text{SC}}|^2} \right) = L_{\text{ref}} - L_{\text{SC}}
\]

- IL along the space integrated in frequency

Reduction of the reflected field depending on the reflection angle
Insertion loss results

- Modified Insertion Loss to study reflection

\[ IL \ [\text{dB}] = 10 \log_{10} \left( \frac{|P_{\text{ref}}|}{|P_{\text{SC}}|} \right)^2 = L_{\text{ref}} - L_{\text{SC}} \]

- IL in frequency bands integrated in ROI

Reduction of the reflected field due to diffusion
Higher in propagative bands
Lower in the BG
Difussion coefficient

- Near to far field transformation
  - Projection of the pressure in near to field to infinity
    - Essentially it is a spatial Fourier transform
  - Angular information
    \[ p_s(r) = -\frac{jk}{8\pi^2}e^{-jk(r+r_0)sinc\left(\frac{kb}{r}\right)[\cos \theta + 1]} \int_{-a}^{a} R(r_s)e^{jkx_s \sin \theta} dx \]

Reflection from a flat rigid surface

• Diffusion coefficient

\[ \delta = \frac{\left( \sum_{i=1}^{n} I_i \right)^2 - \sum_{i=1}^{n} I_i^2}{(n-1) \sum_{i=1}^{n} I_i^2} \]

• Normalized diffusion coefficient

\[ \delta_n = \frac{\delta - \delta_{\text{ref}}}{1 - \delta_{\text{ref}}} \]

• Quantification of the type of reflection

  ◦ Specular \( \delta = 0 \)

  ◦ Diffuse \( \delta = 1 \)

ISO 17497-2:2012
Acoustics -- Sound-scattering properties of surfaces -- Part 2:
Measurement of the directional diffusion coefficient in a free field
Difussion coefficient results

- Far field results
- Normalized diffusion coefficient

\[
\delta_n = \frac{\delta - \delta_{\text{ref}}}{1 - \delta_{\text{ref}}}
\]

### Sonic Crystal - TL

- (a) \( f = 360 \text{ kHz} \)
- (b) \( f = 500 \text{ kHz} \)
- (c) \( f = 660 \text{ kHz} \)

### Reference (flat reflector)

- (d) \( f = 360 \text{ kHz} \)
- (e) \( f = 500 \text{ kHz} \)
- (f) \( f = 660 \text{ kHz} \)

- (g) Graph showing diffusion coefficient variation with frequency for different materials (SL, TL, SH, TH).
Sonic Crystals: Fundamentals, characterization and experimental techniques

A. Cebrecos

1Laum, Le Mans Université, CNRS, Av. O. Messiaen, 72085, Le Mans

Collaborators in this work:

LAUM: J.P. Groby, V. Romero-García
UPV: N. Jiménez, V. Sánchez-Morcillo, L.M. García-Raffi
UCB: M. Hussein, D. Krattiger
Positive, zero and negative diffraction.

• Focusing of waves using finite SC’s:
  - Curvature of the wave front
    \[ \vec{k} = (k_x, k_y) \]
  - Character of the incident wave
    - Plane wave
    - Point source
    - Sound beam (Gaussian beam)
      - Medium A
      - SC
      - Medium A

\[ D = 2a \]
\[ D = 8a \]
Focusing of waves using finite SC’s:

- Interplay between beam and periodic media
  - Band structure and Isofrequency contours (PWE)

- Spatial spectrum of the incident beam

Medium A | SC | Medium A
---|---|---
Linear source

$$a = 5.25 \text{ mm}$$
$$r = 0.8 \text{ mm}$$

Band structure

Isofrequency contour

D = 2a

D = 8a

Source angular spectrum

Extended BZ

DISPERSION. GROUP AND PHASE VELOCITIES
Results

• Spatial dispersion completely parabolic

\[ (d_4 k_y^4 \rightarrow 0) \]

Accumulated phase shift

\[ \Delta \varphi (k_x) = -d_2 k_y^2 L \]

\[ d_2 L + z_f = 0 \]

\[ z_f = \alpha L \left( \frac{f^2}{\Delta \omega^2} \right) \]

Focusing distance

DISPERSION. GROUP AND PHASE VELOCITIES

\[ \text{Propagation of sound beams behind sonic crystals} \]

V. J. Sánchez-Morcillo, K. Staliunas, V. Espinosa, I. Pérez-Arjona, J. Redondo, and E. Soliveres