DENORMS TRAINING SCHOOL 3 “Experimental techniques for acoustic porous materials and metamaterials”, 4th-6th December 2017, Le Mans

Ultrasonic characterisation of porous materials

Philippe Leclaire
ISAT – Institut Supérieur de l’Automobile et des Transports
DRIVE EA1859 – Département de Recherche en Ingénierie des Véhicules pour l’Environnement
Nevers, Université de Bourgogne, France
1. Acoustic wave propagation in air

2. Viscous boundary layer effects in ducts

3. Wave propagation in porous and perforated media in the rigid frame approximation

4. Ultrasonic characterization of porous (and perforated) materials

5. Current work and other applications

6. Highlights

7. Appendix
1. Acoustic wave propagation in air
“A wave is a perturbation that propagates”

Examples:
- Light/Electromagnetic waves
- Seismic waves
- Acoustics waves

Acoustic waves need matter to propagate

“Perturbation” implies that there is an equilibrium state or a medium at rest

Acoustic waves

Mechanical disturbance of matter (liquid, solid, gas), that propagates.

In linear acoustics, the total pressure is written:

\[ P_{Tot} = P_0 + p \]

\( p \): small acoustic disturbance of the static pressure \( P_0 \).

For a sound wave, the propagation medium is air.

\( \text{dV, dx, dy, dz} \)

(*Animation courtesy of Dr. Dan Russell, Penn State University)

Boundary effects to be clarified!
Linearized wave equation

\[ \Delta p - \frac{1}{c_\varphi^2} \frac{\partial^2 p}{\partial t^2} = 0 \]

Source(s)

\[ \Delta p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \]

Spatial variations of the sound pressure

\[ \frac{\partial^2 p}{\partial t^2} \]

Time variations of the sound pressure

\[ c_\varphi \]

Phase velocity. Connects time and space

General integral (solution) of the wave equation (1D case)

Harmonic solutions

\[ s = a e^{i(\omega t - kx)} + b e^{i(\omega t + kx)} \]

Forward going wave along the x axis

Backward going wave along the negative x axis

If \( b = 0 \), solution only forward going
2. Viscous boundary effects in ducts
Harmonic sound waves in a duct – Boundary effects

University of Southampton - ISVR
Frequency and viscous skin depths – Orders of magnitudes

The boundary layer thickness depends on frequency:

\[ \delta = \sqrt{\frac{2\eta}{\rho_0 \omega}} \]  
(for a flat plate)

<table>
<thead>
<tr>
<th>( f )</th>
<th>20 Hz</th>
<th>5 kHz</th>
<th>20 kHz</th>
<th>500 kHz</th>
<th>1 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>481 μm</td>
<td>30.4 μm</td>
<td>15.2 μm</td>
<td>3.04 μm</td>
<td>2.2 μm</td>
</tr>
</tbody>
</table>

For a pore or perforation of radius 100 μm, the viscous skin depth in the ultrasonic range at 500 kHz is of the order of 3% of the pore radius.
3. Wave propagation in porous and perforated media in the rigid frame approximation
"Low Frequency" Biot based models

1956, 1962 : Biot (Sinuosity and structural shape factors)
1982 : Attenborough (circular functions, q)
1986 : Johnson, Koplik, Dashen \( k_0, \alpha_\infty, \Lambda \)
1991 : Champoux, Allard \( \Lambda' \)
1993 : Pride et al. \( b \rightarrow '' \alpha_0'' \)
1997 : Lafarge et al. \( k'_0 \)

1970' \( \rightarrow ... \) Other models : (Zwikker Kosten, Delany Bazley, Miki, Voronina, Yamamoto, Turgut, Horoshenkov, Stinson, Wilson, etc...)

“High frequency” Scattering models

\(~ 1950' \rightarrow -...\)
Scattering theories,
Resonant scattering theories...
What happens during the propagation? (at all frequencies)

• Inertial coupling:
  - Kinetic energy exchange
  - Momentum exchange
  \( \phi, \alpha_\infty, \rho_0, \rho_s \)

• Elastic coupling
  - Potential deformation energy exchange
  \( \phi, E, \mu, P_0, C \)

• Visco-thermal interactions
  - Viscous frictions and thermal losses in boundary layers
  \( \phi, \sigma, \Lambda, \Lambda' \)
Rigid frame approximation

\[ \rho_0 \ll \rho_s \rightarrow \infty \]
\[ \gamma P_0 \ll E, \mu \rightarrow \infty \]

\[ \varepsilon_{ij} = 0, \theta = 0, \quad u = 0, \dot{u} = 0, \ddot{u} = 0 \]

**Biot's equations of poroelasticity**

\[
\begin{align*}
\sigma_{ij} &= 2 \mu \varepsilon_{ij} + \lambda \theta \delta_{ij} + C \theta e \\
\sigma &= Ke + C \theta e
\end{align*}
\]

**Equations of motion**

\[
\begin{align*}
\sigma_{i,j} &= \rho_{11} \ddot{u}_i + \rho_{12} \ddot{U} \\
\sigma_{,j} &= \rho_{21} \ddot{u}_j + \rho_{22} \ddot{U}
\end{align*}
\]

Not accounting for dissipation potential

**Wave equation (1D) written in \( U \)**

\[ K \frac{\partial^2 U}{\partial x^2} = \alpha_\infty \phi \rho_0 \dot{U} \]

\[ K = \phi K_e \]
For rigid frame approximation (simplification for limp frame also possible)

IF
\[ \rho_0 \ll \rho_s \rightarrow \infty \]
\[ \gamma P_0 \ll E, \mu, \rightarrow \infty \]

THEN

Wave equation
\[ \Delta p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \]

Complex phase velocity of the wave
\[ c_\phi(\omega) = \sqrt{\frac{K_e(\omega)}{\rho_e(\omega)}} \]

Complex wavenumber
\[ k(\omega) = \frac{\omega}{c_\phi(\omega)} \]

Wave velocity and attenuation
\[ c(\omega) = \frac{\omega}{k_R(\omega)} \quad \text{and} \quad \alpha(\omega) = k_I(\omega) \]

Characteristic impedance, Surface impedance, TMM, etc…
Johnson-Champoux-Allard (JCA) model

\[ \rho_e(\omega) = \alpha_\infty \rho_0 \left(1 - j \frac{\phi}{\omega \rho_0 \alpha_\infty} \sigma G(\omega) \right), \]

\[ K_e(\omega) = \frac{\gamma P_0}{\gamma - (\gamma - 1) \left(1 - j \frac{H_p}{2 \omega} G_p(\omega) \right)^{-1}} \]

with

\[ G(\omega) = \sqrt{1 + \frac{j \omega}{H}}, \quad H = \frac{\sigma^2 \phi^2 \Lambda^2}{4 \eta \rho_0 \alpha_\infty^2}, \]

\[ G_p(\omega) = \sqrt{1 + \frac{j \omega}{H_p}}, \quad H_p = \frac{16 \eta}{Pr \Lambda^2 \rho_0} \]

5 physical parameters: \( \phi, \sigma, \alpha_\infty, \Lambda, \Lambda' \)

**REMARK:**
- JCA model or any other model is not related to rigid or limp frame approximation
- Any model (JCA, Attenborough, Lafarge et al., etc…) applies to poroelastic materials

**PROVIDED** \( \lambda \gg d \)
HF asymptotic expansion of JCA model

\( \alpha_\infty, \Lambda, \Lambda' \) are defined in the high frequency limit for \( \delta \to 0 \)

- Inviscid fluid (ex: superfluid He, air)
- Usual fluid in the HF limit

- Ultrasonic methods in air
- Direct experimental access to \( \alpha_\infty, \Lambda, \Lambda' \)

In the high frequency limit:

\[
\rho_e(\omega) = \alpha_{\infty,0} \rho_0 \left( 1 - j \frac{\phi}{\omega \rho_0 \alpha_\infty} \sigma G(\omega) \right),
\]

\[
K_e(\omega) = \gamma P_0 \frac{\gamma P_0}{\gamma - (\gamma - 1) \left( 1 - j \frac{H_p}{2 \omega} G_p(\omega) \right)^{-1}}.
\]

Other asymptotic expansion (LF or HF): Later if necessary

\[
\frac{c_\varphi(\omega)}{\sqrt{\rho_e(\omega)}} \to \frac{c_0}{\sqrt{\alpha_\infty}}
\]

\[
\alpha(\omega) \to \text{Const} \times \sqrt{\omega}
\]
Dispersion curves

Diffusion domain

Attenuation increases with frequency but attenuation per cycles decreases

\[ c(\omega) = \frac{\omega}{k_R(\omega)} \]

\[ \omega_c = \frac{\phi \sigma}{\alpha_{\infty} \rho_0} \]

\[ \alpha(\omega) = k_I(\omega) \]
Definitions of $\alpha_\infty$, $\Lambda$, $\Lambda'$

Porosity $\phi$

Flow resistivity $\sigma$

Viscous characteristic length $\Lambda$

Thermal characteristic length $\Lambda'$

Image of ATF 1 mm

Tortuosity $\alpha_\infty$
Definitions of $\alpha_\infty$, $\Lambda$, $\Lambda'$

$\alpha_\infty = q^2$

$V_T$: macroscopic volume of porous aggregate and containing a volume $V$ of fluid

$\alpha_\infty = \frac{1}{V} \iiint_V v^2 dV \left( \frac{1}{V} \iiint_V v dV \right)^2$,

$\Lambda = 2 \iiint_V v^2 dV \iiint_S v_w^2 dS$,

$\Lambda' = 2 \iiint_S dV \iiint_S dS$,

$S$: (Cumulative) surface of the pore walls

$\Lambda'$ related to specific area

$\Lambda' \geq \Lambda$
4. Ultrasonic characterization of porous (and perforated) materials
Ultrasonic waves in porous media: Contributions

Johnson, Plona, Scala, Pasierb, Kojima 1982, 1994
Singer et al., 1984

Nagy, 1990

Allard, Castagnède, Henry, Lauriks, 1994
Leclaire et al. 1996
Brown, Melon, Castagnède, 1996
Catagnède et al., 1996

Nagy, Johnson, 1996
Ayrault, Moussatov, Catagnède, Lafarge, 1999
Moussatov, Ayrault, Castagnède, 2001

Fellah, Berger, Lauriks, Depollier et al., 2003
Fellah et al., 2007

Umnova, Shin, Attenborough, Cummings, 2005

Acoustic probing of porous media with superfluid He

Ultrasonic waves in air-filled porous rocks
Measurement of tortuosity using airborne ultrasonic waves
Measurement of \( \alpha_\infty, \Lambda, \Lambda' \) with air-coupled capacitive transducers

Pressure dependent ultrasonic propagation

Reflected waves, oblique incidence
Time domain, Inverse methods \( \phi, \alpha_\infty \)
Laser spark source and microphones
Tortuosity from reflection and transmission
Analytical expressions of transmitted and reflected ultrasonic waves
Ultrasonic characterisation of porous media - The Qδ method

1\textsuperscript{st} order HF expansion of the density

\[ \rho_e(\omega) = \alpha_\infty \rho_0 \left( 1 - j \frac{\phi \sigma}{\omega \rho_0 \alpha_\infty} \sqrt{1 + j \omega \frac{4 \eta \rho_0 \alpha_\infty^2}{\sigma^2 \phi^2 \Lambda^2}} \right) \rightarrow \alpha_\infty \rho_0 \left( 1 - j \frac{\phi \sigma}{\omega \rho_0 \alpha_\infty} \sqrt{j \omega \frac{4 \eta \rho_0 \alpha_\infty^2}{\sigma^2 \phi^2 \Lambda^2}} \right) \]

\[ \rho_e(\omega) \rightarrow \alpha_\infty \rho_0 \left( 1 - j \sqrt{\frac{4 \eta}{\omega \rho_0 \Lambda^2}} \left( \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right) \right) = \alpha_\infty \rho_0 \left( 1 - j \frac{1}{\Lambda} \sqrt{\frac{2 \eta}{\omega \rho_0}} (1 + j) \right) \]

Since

\[ \delta = \sqrt{\frac{2 \eta}{\rho_0 \omega}} \]

\[ \rho_e(\omega) \rightarrow \alpha_\infty \rho_0 \left( 1 + \frac{\delta}{\Lambda} (1 - j) \right) \]

Similarly for the bulk modulus (see APPENDIX)

\[ K_e(\omega) \rightarrow \frac{\gamma P_0}{\gamma - (\gamma - 1) \left( 1 + \frac{\delta}{\Lambda' \sqrt{Pr}} (j - 1) \right)} \]
Ultrasonic characterisation of porous media - The Qδ method

1\textsuperscript{st} order HF expansion of the wavenumber $k$

$$\frac{K_e(\omega)}{\rho_e(\omega)} = c_\varphi^2 \rightarrow \frac{\gamma P_0}{\left(\gamma - (\gamma - 1)\left(1 + \frac{\delta}{\Lambda' \sqrt{Pr}}(j - 1)\right)\right)\alpha_\infty \rho_0 \left(1 + \frac{\delta}{\Lambda}(1 - j)\right)}$$

and

$$k(\omega) = \frac{\omega}{c_\varphi} \rightarrow \omega \frac{\sqrt{\alpha_\infty}}{c_0} \left[1 + \frac{(1 - j)\delta}{2} \left(\frac{1}{\Lambda} + \frac{(\gamma - 1)}{\Lambda' \sqrt{Pr}}\right)\right]$$

By using the attenuation per cycle

$$\frac{1}{Q} = \frac{-2 \text{Im}[k]}{\text{Re}[k]}$$

where $Q$ is the quality factor

It is easily seen that (see APPENDIX)

$$\lim_{\omega \to \infty} Q\delta = \left(\frac{1}{\Lambda} + \frac{(\gamma - 1)}{\Lambda' \sqrt{Pr}}\right)$$

NB: We can define an "equivalent length »

$$L_\text{eq} = \left(\frac{1}{\Lambda} + \frac{(\gamma - 1)}{\Lambda' \sqrt{Pr}}\right)^{-1}$$
Recall Dispersion curves

No strong dispersion?

Diffusion regime

Attenuation increases with frequency but attenuation per cycles decreases

\[ c(\omega) = \frac{\omega}{k_R(\omega)} \]

\[ \omega_c = \frac{\phi \sigma}{\alpha_\infty \rho_0} \]

\[ \alpha(\omega) = k_I(\omega) \]
The Qδ method - Experiment

(Leclaire, Kelders, Lauriks, Melon, Brown, Castagnède, 1996)

Broadband (0.1 - 1 MHz) air-coupled capacitive transducers with mylar membrane (Developed by D.A. Hutchins, D. W. Schindel et al.)

Simultaneous measurement of \( \Lambda, \Lambda' \)

Simultaneous measurement of \( \Lambda, \Lambda' \)

He, air

transducer

transducer

Porous material

Amplifier

Amplifier

Function generator

Oscilloscope
Exemple of time signal without and with porous material

Corresponding amplitudes of the spectra
Fréquence (kHz)

0 100 200 300 400 500 600 700 800 900 1000

\[ Q \delta (\mu m) \]

0 50 100 150 200 250

Air

Hélium

\[ \lim_{\omega \to \infty} (Q\delta)_{air} = \left[ \frac{1}{\Lambda} + \frac{\gamma_{air} - 1}{B_{air} \Lambda'} \right]^{-1} \]

\[ \lim_{\omega \to \infty} (Q\delta)_{He} = \left[ \frac{1}{\Lambda} + \frac{\gamma_{He} - 1}{B_{He} \Lambda'} \right]^{-1} \]

HF limit before scattering

Biot based models

Scattering models

LF

\( f_c \)

HF

\( \delta \ll h \)

LF

\( d \ll \lambda \)

HF

\( d \approx \lambda \)
Ultrasonic characterisation of porous media - The $n^2$ method

(Castagnède, Melon et al., 1996)

From

$$k(\omega) \rightarrow \omega \frac{\sqrt{\alpha_c}}{c_0} \left[ 1 + \left( 1 - j \right) \delta \right] \quad \text{Recall} \quad L_{eq} = \left( \frac{1}{\Lambda} + \frac{(\gamma - 1)}{\Lambda' \sqrt{Pr}} \right)^{-1}$$

The refraction index $n$ squared is

$$n^2 = \left( \frac{c_0}{c} \right)^2 \rightarrow \alpha_\infty \left[ 1 + \frac{\delta}{L_{eq}} \right]$$

Or

$$n^2 \rightarrow \alpha_\infty \left[ 1 + \sqrt{\frac{2\eta}{\rho_0 \omega L_{eq}^{-1}}} \right]$$
The $n^2$ method

Same experimental setup involving He and air

Simultaneous measurement of

$\alpha_\infty, \Lambda, \Lambda'$

Intercept

Slopes

Hélium

Air

$\log_{10}(n)$

$1.04$ $1.08$ $1.12$ $1.16$

$0$ $1e-3$ $2e-3$ $3e-3$ $4e-3$

$f^{-1/2} (\text{Hz}^{-1/2})$
Measurement of tortuosity and porosity from ultrasonic reflected waves


Reflection coefficient in the time domain

\[ r(t, \theta) = \frac{\alpha_\infty \cos \theta - \phi \sqrt{\alpha_\infty - \sin^2 \theta}}{\alpha_\infty \cos \theta + \phi \sqrt{\alpha_\infty - \sin^2 \theta}} \delta(t). \]

At least 2 measurements at different angles are necessary

\[ \alpha_\infty = \frac{\left(\frac{(1-r_2)(1+r_1)\cos \theta_2}{(1+r_2)(1-r_1)\cos \theta_1}\right)^2 \sin^2 \theta_1 - \sin^2 \theta_2}{\left(\frac{(1-r_2)(1+r_1)\cos \theta_2}{(1+r_2)(1-r_1)\cos \theta_1}\right)^2 - 1} \]

Porosity is deduced from tortuosity

\[ \phi = \frac{\alpha_\infty (1-r_i)\cos \theta_i}{(1+r_i)\sqrt{\alpha_\infty - \sin^2 \theta_i}}, \quad i = 1, 2. \]

The reflection coefficient is obtained from the amplitude of the time signals

Static pressure variations

Moussatov, Ayrault, Castagnède, Ultrasonics, 2001

\[
\lim_{\omega \to \infty} n^2_r(\omega, P_0) = \xi_\infty \left(1 + \sqrt{\frac{2\eta RT}{M} \frac{1}{l_{vt}} \frac{1}{\sqrt{\omega P_0}}} \right),
\]

(6)

\[
\lim_{\omega \to \infty} |\ln|T(\omega, P_0)|| = \ln(\varepsilon) + \left(\frac{\eta L}{2\gamma l_{vt}}\right)^2 \sqrt{\frac{\omega}{P_0}},
\]

(7)

with

\[
l_{vt} = \left(1 + \frac{\gamma - 1}{\sqrt{Pr} A'}\right)^{-1} \text{ and } \varepsilon = \left(1 + \frac{\phi}{\sqrt{\xi_\infty}}\right)^2 \frac{4\phi}{\sqrt{\xi_\infty}}.
\]

(8)
Laser generated spark source and microphones as receivers

Umnova, Shin, Attenborough, Cummings, 2005

Reflected and transmitted wave

Determination of porosity and tortuosity
Analytical expressions as functions of effective density and bulk modulus

\[ \tilde{\rho} = \frac{\rho^{[1]}}{\rho^{[0]}} = \frac{\alpha_\infty}{\phi} \left( 1 + \frac{2}{\Lambda} \sqrt{\frac{i\eta}{\omega \rho_f}} \right), \]

\[ \tilde{K} = \frac{K^{[1]}}{K^{[0]}} = \frac{1}{\phi} \left( 1 + \frac{2(1 - \gamma)}{\Lambda'} \sqrt{\frac{i\eta}{\omega \text{Pr} \rho_f}} \right), \]  \quad (6)

\[ \tilde{\Lambda} = \sqrt{\frac{2\eta}{\omega \rho_f}} \left( \frac{\text{Re}(\tilde{\rho}) - \text{Im}(\tilde{\rho})}{\text{Im}(\tilde{\rho})} \right), \]

\[ \tilde{\Lambda}' = (1 - \gamma) \sqrt{\frac{2\eta}{\omega \text{Pr} \rho_f}} \left( \frac{\text{Re}(\tilde{K}) - \text{Im}(\tilde{K})}{\text{Im}(\tilde{K})} \right). \]  \quad (7)

Then, \( \phi \) is recovered from \( \| \tilde{K} \| \) via

\[ \phi = \frac{1}{\| \tilde{K} \|} \left( 1 + \frac{2(1 - \gamma)}{\Lambda'} \sqrt{\frac{2\eta}{\omega \text{Pr} \rho_f} + \frac{4(1 - \gamma)^2\eta}{\Lambda'^2\omega \text{Pr} \rho_f}} \right)^{1/2}, \]  \quad (8)

and finally \( \alpha_\infty \) is recovered from \( \| \tilde{\rho} \| \) by means of

\[ \alpha_\infty = \phi \| \tilde{\rho} \| \left( 1 + \frac{2}{\Lambda} \sqrt{\frac{2\eta}{\omega \rho_f} + \frac{4\eta}{\Lambda^2\omega \rho_f}} \right)^{-1/2}. \]  \quad (9)


\( \tilde{\rho} \) and \( \tilde{K} \) can be recovered from experimental data via reflection and transmission coefficients.

Method inspired by and complementary to the method by:


Transmission experiments.

Experimental setup is the same as previous studies.
5. Current work and other applications
Intercorrelation method

ULTRAN Air coupled ultrasonic Transducer
Narrowband
(50, 100, 250 kHz)

More energy at a given frequency

\[ C_{xy}(\tau) = \lim_{T \to \infty} \int_T x(t) y(t + \tau) dt \]

x: with Sample, y: without sample

Max of this function provides time delay ⇒ Provides the tortuosity
Other application: Determination of Kinematic porosity / DE porosity


Great difference between theoretical and experimental results, although “reasonable” confidence in JCA parameters measurements

A problem was identified:

Porosity seems greatly overestimated in JCA model

Suspicion that materials contain microcavities or Dead-ends
Other materials with suspected DE pores

- Aluminum foam
- Sunflower stalk and chitosan
- Porous concrete
- Porous concrete with hemp or flax particles
- … etc
- e.g.: Consolidated Materials with low porosity
- Perforated plates with surface dead-ends
- Suspicion confirmed!
Determination of Kinematic porosity / DE porosity

\[ \phi_{Tot} = \phi + \phi_C \]

\[ \phi = \phi_k + \phi_{DE} \]

\( \phi_k \) is the porosity actually “seen” by the wave \( \equiv \) use of ultrasonic measurements

\( \phi \) is the open porosity \( \equiv \) measured by classical non acoustic methods

Ex: Boyle’s law based method, Pressure mass method by Salissou and Panneton., 2007, Comparison of air volume by Leclaire et al., 2003
Determination of kinematic porosity / DE porosity

Much better agreement with porosity correction accounting for DE pores

\[ \phi = 86.6\% \pm 2\% \]

Panneton method

\[ \phi_k = 72\% \pm 4\% \]

Various US methods

Fellah’s method

\[ \phi_{DE} = \phi - \phi_k \]

\[ = 14.6\% \pm 4\% \]

Deduced

NB:
- Structural resonance discarded
- Double porosity effect non accounted for
Determination of kinematic porosity / DE porosity


LF measurements
HF model asymptote

Black line is least square fit of experimental data

In log-log scale, we see linear experimental behaviour

HF behaviour is a horizontal line

The intercept provides the cutoff frequency which is also:

\[ f_c = \sigma \frac{\phi_k}{2\pi\alpha_\infty \rho_0} \]
Determination of kinematic porosity / DE porosity


HF measurements
HF model asymptote

Moussatov’s asymptotic expansion

$$\lim_{\omega \to \infty} \ln |T(\omega)| = \ln(\varepsilon) + \sqrt{\omega} \left( \frac{\eta \alpha_{\infty}}{2\gamma P_0 L_{eq}} \right),$$

with parameter $\varepsilon$ and equivalent length $L_{eq}$ given by

$$\varepsilon = \frac{(1 + \phi_B/\sqrt{\alpha_{\infty}})^2}{4\phi_B/\sqrt{\alpha_{\infty}}},$$

$$L_{eq} = \left( \frac{1}{\Lambda} + \frac{\gamma - 1}{\sqrt{Pr\Lambda'}} \right)^{-1},$$
- Acoustic wave propagation in air, viscous boundary layer effects in ducts, wave propagation in porous and perforated media
  - Rigid frame approximation
  - Johnson-Champoux-Allard model

- Ultrasonic characterization of porous (and perforated) materials
  - Broadband transmission in porous media saturated by different gases: the $Q\delta$ and $n^2$ methods
  - Measurement of tortuosity and porosity from ultrasonic reflected waves
  - Static pressure variations
  - Laser generated spark source and microphones as receivers
  - Analytical expressions as functions of effective density and bulk modulus
  - Narrowband transmission and intercorrelation method for tortuosity
  - Application of ultrasonic method for the measurement of kinetic and Dead-End porosity
Thank you for your attention
1st order HF expansion of the bulk modulus

\[ K_e(\omega) = \frac{\gamma P_0}{\gamma - (\gamma - 1) \left( 1 - \frac{j H_p}{2 \omega} \sqrt{1 + \frac{j \omega}{H_p}} \right)^{-1}} \rightarrow \frac{\gamma P_0}{\gamma - (\gamma - 1) \left( 1 + \frac{j H_p}{2 \omega} \sqrt{\frac{j \omega}{H_p}} \right)} \]

\[ \rightarrow \frac{\gamma P_0}{\gamma - (\gamma - 1) \left( 1 + \frac{j}{2} \sqrt{\frac{H_p}{\omega}} \left( \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \right) \right)} = \frac{\gamma P_0}{\gamma - (\gamma - 1) \left( 1 + j \frac{H_p}{8 \omega} (1 + j) \right)} \]

Since \[ H_p = \frac{16 \eta}{Pr \Lambda^2 \rho_0}, \quad \delta = \sqrt{\frac{2 \eta}{\rho_0 \omega}} \]

\[ K_e(\omega) \rightarrow \frac{\gamma P_0}{\gamma - (\gamma - 1) \left( 1 + j \frac{1}{\Lambda' \sqrt{Pr}} \sqrt{\frac{2 \eta}{\rho_0 \omega}} (1 + j) \right)} = \frac{\gamma P_0}{\gamma - (\gamma - 1) \left( 1 + \frac{\delta}{\Lambda' \sqrt{Pr}} (j - 1) \right)} \]
1st order HF expansion of the wavenumber $k$

$$K_e(\omega) = c_\varphi^2 ightarrow \frac{\gamma P_0}{\rho_e(\omega)} = \frac{\gamma (j-1)(1+\frac{\delta}{\Lambda'\sqrt{Pr}}(j-1))}{\alpha_\infty \rho_0(1+\frac{\delta}{\Lambda}(1-j))}$$

$$c_\varphi^2 \rightarrow c_0^2 = \frac{1}{\alpha_\infty \left(\gamma - (\gamma - 1)(1+\frac{\delta}{\Lambda'\sqrt{Pr}}(j-1)) + \frac{\delta}{\Lambda}(1-j)\right)}$$

$$c_\varphi^2 \rightarrow c_0^2 = \frac{1}{\alpha_\infty \left(\gamma - (\gamma - 1)(1+\frac{\delta}{\Lambda'\sqrt{Pr}}(j-1)) + \frac{\delta}{\Lambda}(1-j)\right)}$$

$$c_\varphi^2 \rightarrow c_0^2 = \frac{1}{\alpha_\infty \left(1+\left(\frac{\delta}{\Lambda'\sqrt{Pr}}(1-j)+\frac{\delta}{\Lambda}(1-j)\right)\right)} = \frac{1}{\alpha_\infty \left(1+\delta(1-j)\left(\frac{1}{\Lambda} + \frac{\gamma-1}{\Lambda'\sqrt{Pr}}\right)\right)}$$
$1^{st}$ order HF expansion of the wavenumber $k$ continued and limit of Qδ

$$c^2_{\phi} \rightarrow c^2_0 \frac{1}{\alpha_\infty \left( 1 + \delta (1-j) \left( \frac{1}{\Lambda} + \frac{\gamma-1}{\Lambda' \sqrt{Pr}} \right) \right)}$$

$$k(\omega) = \frac{\omega}{c_{\phi}} \rightarrow \frac{\omega \sqrt{\alpha_\infty}}{c_0} \sqrt{1 + (1-j)\delta \left( \frac{1}{\Lambda} + \frac{(\gamma-1)}{\Lambda' \sqrt{Pr}} \right)}$$

$$k(\omega) \rightarrow \omega \frac{\sqrt{\alpha_\infty}}{c_0} \left[ 1 + \frac{(1-j)\delta}{2} \left( \frac{1}{\Lambda} + \frac{(\gamma-1)}{\Lambda' \sqrt{Pr}} \right) \right]$$

$$\frac{1}{Q} = \frac{-2 \text{Im}[k]}{\text{Re}[k]} = \delta \left( \frac{1}{\Lambda} + \frac{(\gamma-1)}{\Lambda' \sqrt{Pr}} \right) \rightarrow \delta \left( \frac{1}{\Lambda} + \frac{(\gamma-1)}{\Lambda' \sqrt{Pr}} \right)$$
$1^{st}$ order LF expansion of the wavenumber $k$ and of expression for velocity and attenuation

$$\rho_e(\omega) \rightarrow \alpha_\infty \rho_0 \left(1 - j \frac{\phi_k}{\omega \rho_0 \alpha_\infty} \sigma\right) = -j \frac{\phi_k}{\omega} \alpha_\infty \rho_0 \sigma,$$

$$K_e(\omega) \rightarrow \frac{\gamma P_0}{\gamma - (\gamma - 1) \left(1 - \frac{H_p}{\gamma 2 \omega} \right)^{-1}} \rightarrow \frac{\gamma P_0}{\gamma - (\gamma - 1) \left(\frac{2 \omega}{H_p}\right)} \rightarrow P_0$$

$$k(\omega) = \omega \sqrt{\frac{\rho_e}{K_e}} \rightarrow \omega \sqrt{-j \frac{\phi_k}{\omega P_0} \alpha_\infty \rho_0 \sigma} = \omega \sqrt{\frac{\phi_k}{\omega P_0} \alpha_\infty \rho_0 \sigma \left(\frac{1}{\sqrt{2}} - j \frac{1}{\sqrt{2}}\right)}$$

$$c = \frac{\omega}{\text{Re}[k]} \rightarrow \sqrt{2 \omega \frac{P_0}{\rho_0 \phi_k \alpha_\infty \sigma}}$$

$$\alpha = -\text{Im}[k] \rightarrow \sqrt{\omega \frac{\phi_k \rho_0 \alpha_\infty \sigma}{2P_0}}$$